A quantum-well laser diode equivalent circuit model with improved stability

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ABSTRACT

An equivalent circuit model for quantum-well laser diodes is presented. The circuit model is derived from modified rate equations and eliminates unstability in the conventional model. SPICE dimulations show the improved stability.

I. INTRODUCTION

Equivalent circuit models for laser diodes(LDs) play an important role for designing LD driver circuits and packages. Several equivalent circuit models have been reported for double heterostructure [1,2] and quantum-well(QW) [3-5] LDs. Kan and Lau [3] implemented the equivalent model using a simple RLC circuit, but their model is not sufficient for large signal responses of QW LD. Lu *et al.* reported simulation results of L-I curve and transient pulse response in [4] and small-signal response in [5] for QW LD using an equivalent circuit model. We find, however, HSPICE simulation based on this approach shows unstable behaviors. Such unstability greatly reduces the utility of an LD equivalent circuit. In order to eliminate this unstability problem, the circuit model based on the transformed rate equations [6] is implemented and HSPICE simulations are performed. In this letter, simulation results are presented for both conventional and improved models and their results are compared..

II. IMPROVED CIRCUIT MODEL

The dynamics of QW LD can be described by the rate equations proposed by Nagarajan *et al.* [7,8] as follow:

$$\frac{dN_{SCH}}{dt} = \frac{I}{qV_{SCH}} - \frac{N_{SCH}}{\tau_r} - \frac{N_{SCH}}{\tau_{nb}} + \frac{N_W(V_W/V_{SCH})}{\tau_e} \quad (1)$$

$$\frac{dN_W}{dt} = \frac{N_{SCH}(V_{SCH}/V_W)}{\tau_r} - \frac{N_W}{\tau_n} - \frac{N_W}{\tau_{nr}} - \frac{N_W}{\tau_e} - g_o(N_o - N_W)(1-\varepsilon S)\cdot S \quad (2)$$

$$\frac{dS}{dt} = \Gamma g_o(N_o - N_W)(1-\varepsilon S)\cdot S - \frac{S}{\tau_p} + \Gamma\beta\frac{N_W}{\tau_n} \quad (3)$$

Here, I is the injected current and S is the photon density. τ_n, τ_{nr},and τ_{nb} are the radiative recombination lifetime, the non-radiative recombination lifetime in the QW, and the total recombination lifetime in the SCH, respectively. τ_p, τ_r,and τ_e are the photon lifetime, the carrier transport time across the SCH region, and the carrier thermionic emission time, respectively. Γ, β, g_o, N_o, and ε have the usual meaning. V_W and V_{SCH} are volume of QW and the SCH region, repectively. N_W and N_{SCH} are carrier density of QW and the SCH region, repectively.

Above equations can be transformed into the following equations.

$$I = \tau_r \frac{dI_{SCH}}{dt} + \left(1+\frac{\tau_r}{\tau_{nb}}\right) \cdot I_{SCH} - I_{FB} \quad (4)$$

$$I_{SCH} - I_{FB} \quad \tau_n \frac{dI_W}{dt} + I_W \cdot (1+\frac{\tau_n}{\tau_{nr}} + I_{gain} \quad (5)$$

$$I_{gain} + \beta I_W = \frac{qV_W}{\Gamma} \cdot \frac{dS}{dt} + \frac{qV_W}{\Gamma} \quad \frac{S}{\tau_p} \quad (6)$$

Here, I_{SCH} represents $qV_{SCH}N_{SCH}/\tau_r$ and I_W represents $qV_W N_W/\tau_n$. The stimulated emission component I_{gain} is equal to $qV_W g_o(N_W - N_o)(1-\varepsilon S)S$ and the thermionic effect component I_{FB} is equal to $\tau_n I_W/\tau_e$. The optical power per facet can be easily obtained from photon density S with $P_f = S/\vartheta$, where ϑ is the conversion factor. An equivalent circuit model (model I) shown in the Fig. 1 can be constructed based on the above equations.

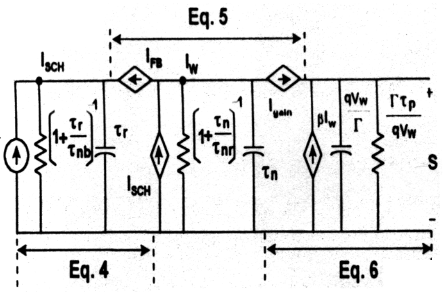

Figure 1: A schematic diagram for LD circuit model I. Arrows indicate corresponding equations given in the text.

In this model, I_{SCH} and I_W are modeled as node voltages. Then, derivative terms such as $\tau_r dI_{SCH}/dt$ in Eq. 4 can be modeled with the current through a capacitor, and constants such as $(1+\tau_r/\tau_{nb})^{-1}$ in Eq. 4 with a resistor. This modeling method is essentially same as that used by Lu *et al.* with exception that node voltages instead of currents are used here. HSPICE simulation using model I, however, shows unstability problems for large-signal transient responses when the bias current is over the threshold current(I_{th}). We believe this is due to two steady-state solutions, only one of which is the true solution as pointed out in [6]. This problem can be eliminated if the photon density can be replaced with a quadratic function, and the carrier density with an exponential function. We use photon density $S = \vartheta(m+\delta)^2$ and $N_W = N_e \exp(V/nV_T)$ as suggested in [6]. δ is used in order to avoid a DC convergence problem and in our case, $\delta = 10^{-10}$ was used. With this, the modified rate equations become

$$I = \tau_r \frac{dI_{SCH}}{dt} + \left(1+\frac{\tau_r}{\tau_{nb}}\right) \cdot I_{SCH} - I_{FB} \quad (7)$$

$$I_{SCH} \quad I_{FB} = I_{D1} + I_{D2} + I_{C1} + I_{C2} \quad (8)$$

$$2\tau_p \frac{dm}{dt} + m \quad I_1 + I_2 \quad (9)$$

where I_{D1}, I_{D2}, I_{C1}, I_{C2}, I_1, and I_2 are as following.

$$I_{D1} \quad \frac{qV_W N_e}{\tau_o}\left[\exp\left(\frac{V_j}{nV_T}\right) \quad \right] \quad 10)$$

$$I_{D2} = \frac{qV_W N_e}{2\tau_n}\left[\exp\left(\frac{V_j}{nV_T}\right) - 1 + \frac{2\tau_n}{nV_T}\exp\left(\frac{V_j}{nV_T}\right)\frac{dV_j}{dt}\right] \quad (11)$$

$$I_{C1} = g_o[\tau_o I_{D1} + qV_W(N_e - N_o)] \times (1-\varepsilon\vartheta P_f)\vartheta P_f \quad (12)$$

$$I_{C2} = qV_W N_e \cdot \left(\frac{1}{\tau_n} + \frac{1}{\tau_{nr}}\right) \quad (13)$$

$$I_1 = \Gamma g_o \tau_p \left[\frac{\tau_o}{q V_W} I_{D1} + N_e - N_o \right] \times (1 - \varepsilon \vartheta P_f)(m + \delta) \quad (14)$$

$$I_2 = \frac{\Gamma \beta \tau_p}{\vartheta (m + \delta) \tau_n} \left[\frac{\tau_o}{q V_W} I_{D1} + N_e \right] \quad (15)$$

$$\frac{1}{\tau_o} = \left(\frac{1}{2\tau_n} + \frac{1}{\tau_{nr}} \right) \quad (16)$$

Here, V_j is the applied voltage, V_T is the thermal voltage, and n is the ideality factor. The equivalent circuit model (modelⅡ) obtained from the above equations is shown in Fig. 2.

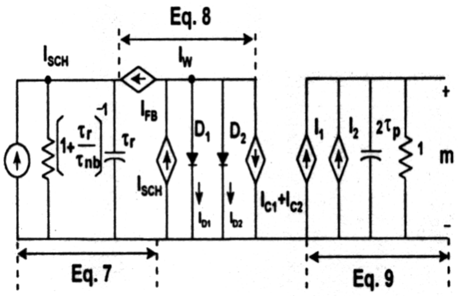

Figure 2: A schematic diagram for LD circuit modelⅡ. Arrows indicate corresponding equations given in the text.

The modelⅡ has resistors, capacitors, voltage-controlled current sources, and diodes. Diodes are used since Eq. 10 corresponds to a typical diode equation and Eq. 11 corresponds to the current through a diode with charge storage effects modeled by the transit time model of HSPICE.

Ⅲ SIMULATION RESULTS

To verify the accuracy and stability of the model proposed above, $In_{0.2}Ga_{0.8}As/GaAs$ SQW (Single Quantum-Well) LD with 1500Å SCH region was simulated using HSPICE. The values for LD parameters were obtained from [8]. With the new equivalent circuit model, L-I curve, large-signal response, and small-signal frequency response simulation can be successfully done without any stability problems. As an example, Fig. 3 shows simulation results of model I (solid line) and modelⅡ (dotted line) for pulse transient responses with the bias current of $1.2I_{th}$. The pulse amplitude is fixed at twice the threshold current. Clearly, the simulation based on model I has a stability problem, resulting in an unrealistic spike. Such unstability problems occur for various bias conditions applied to model I.

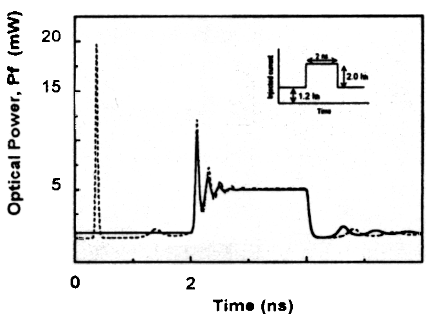

Figure 3. Simulation results for transient response using two different models. (dotted line: model I, solid line: modelⅡ)

In contrast, simulation based on modelⅡ shows no signs of unstability problems during either large-signal transient response and small-signal frequency response simulation at any bias conditions.

Ⅳ. CONCLUSION

An equivalent circuit model for SCH SQW laser diode is derived from the modified rate equations. The model has improved stability and is expected to be useful for LD drive circuitry design and optical interconnect system modeling.

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