# Analysis of QW with Strained Thin Layers for Polarization Insensitive Optical Amplifiers

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A practical approach to achieve the polarization insensitive optical gain at 1.55  $\mu$ m with quantum well (QW) in which thin and highly tensile strained layers are embedded is numerically investigated. The precise band structures and the optical gain characteristics of the structure are calculated. Comparing with the results obtained from the tight-binding model, we concluded that the  $k \cdot p$  method based on the envelop function approximation is still valid for such complicated structures like  $\delta$ -strained quantum wells.

## I. INTRODUCTION

Optical amplifiers are a key component in the long haul optical communication systems. The semiconductor optical amplifiers (SOA's) have great potential as amplifiers, switches and wavelength converters [1]. Besides high gain and high output power, polarization insensitivity is a key feature for most SOA applications. Size inequality in the cross sections of the active region gives rise to polarization sensitive confinement factors for TE and TM mode gains. In the quantum well (QW) structures, the degeneracy of valence bands are making the top valence bands to heavy-hole states. Transitions involving these states have a strong TE component.

In order to achieve the polarization insensitivity, several SOA structures with strained layers have been proposed [1-8]. The main contribution of the strain is to change the states of heavy-hole and light-hole bands. The tensile strain rases the states of the light-hole bands. In  $In_{1-x}Ga_x$  As ternary wells and InGaAsP quaternary barriers grown on InP substrate, the operating wavelength is limited by the critical thickness of the strained layer. The tensile strain below 1 % is not recommended, where the strong band mixing between the heavy-hole and the light-hole states causes higher threshold current densities [9]. There are some papers reporting the 1.55  $\mu$ m polarization insensitive SOA based on QW structures [2,8]. Especially, the QW structure in the reference [8] shows polarization insensitivity for a wide range of wavelength and has the merits of easy growing with delta strained layers embedded in a lattice matched InGaAs/InGaAsP QW [8]. The theoretical analysis on the structure with the tight-binding method has been reported for the case of QW with one GaAs  $\delta$  layer [14]. According to their results, the QW with one GaAs  $\delta$  layer has a serious band mixing at the zone center. This heavy band mixing occurs when the tensile strain is applied below 1 %, making it not possible to have low threshold current density devices. The way to reduce the band mixing effects is inserting more delta layers to the well, which increase the band splits between heavy-hole states and the light-hole states. In this study, we analyzed the QW with one and three  $\delta$  layers. Although we used  $k \cdot p$  method based on the EFA (envelop function approximation), we obtained almost same results with those of [14]. These results indicate that the  $k \cdot p$  method based on the EFA can still explain such complicated structures. With this idea, we analyzed the QW with three  $\delta$  layers. We determine that the band mixing effects are much reduced compare to the case of QW with one  $\delta$  layer by showing the optical gain coefficients of each structure.

#### II. MODEL

For analysis, we used the conventional methods for band structure calculations. The band offsets of each strained layer is obtained from the model solid theory [10]. For the conduction bands, an isotropic parabolic band was used. For valence bands, we used  $4 \times 4$ Luttinger-Kohn Hamiltonian based on the  $k \cdot p$  method [14]. After the unitary transformation of  $4 \times 4$  Luttinger-Kohn Hamiltonian with a uniaxial stress, we obtain [11]

$$H^{\sigma} = \begin{bmatrix} P \pm Q \pm \zeta & \tilde{R} \\ \tilde{R}^{\dagger} & P \mp Q \mp \zeta \end{bmatrix} + V(z)$$

where

$$P = \frac{1}{2} \left(\frac{\hbar^2}{m}\right) \gamma_1 \left(k_x^2 + k_y^2 - \frac{\partial^2}{\partial z^2}\right)$$
$$Q = \frac{1}{2} \left(\frac{\hbar^2}{m}\right) \gamma_2 \left(k_x^2 + k_y^2 + 2\frac{\partial^2}{\partial z^2}\right)$$
$$\widetilde{R} = \frac{\hbar^2}{m} \left[\frac{\sqrt{3}}{2} \overline{\gamma} (k_x^2 + k_y^2) - \sqrt{3} \gamma_3 (k_x^2 + k_y^2)^{1/2} \frac{\partial}{\partial z}\right]$$

where V(z) is the potential for hole,  $\gamma_1, \gamma_2, \gamma_3$  are the Luttinger parameters, *m* is the free electron mass,  $\bar{\gamma} = \frac{1}{2}(\gamma_1 + \gamma_2)$ ,  $\tilde{R}^{\dagger}$  is the adjoint of  $\tilde{R}$ ,  $\sigma = U$  or *L* represents the upper or lower block, and  $\zeta$ , the strain potential is given by

$$\zeta = b \frac{C_{11} + C_{12}}{C_{11}} \epsilon = \frac{1}{2} \delta E_{sh}$$

where b is deformation potential,  $\epsilon$  is the strain,  $C_{11}$  and  $C_{12}$  are the elastic constants, and  $\delta E_{sh}$  is the shear component of the strain.

The upper and lower block envelop functions are given by

$$\psi^{\sigma}_{\mathrm{mk}_{||}}(oldsymbol{r}) = \sum_{
u} g^{(
u)}_{m}(oldsymbol{k}_{||},z) e^{i \mathbf{k}_{||} \cdot oldsymbol{
ho}} |
u >$$

where  $g_m^{(\nu)}(k_{||}, z)$  are the envelop functions,  $\{|\nu\}$  denotes the transformed Bloch basis at the zone center.  $k_{||} = k_x \hat{x} + k_y \hat{y}, \rho = x \hat{x} + y \hat{y}$ , and *m* is the index of the subbands. The notation for  $\nu = 1, 2$  are for the upper block, and 3,4 are for lower block. Then, the envelop functions satisfy the following coupled differential equations

$$\sum_{\nu} \left[ H^{\sigma}_{\nu\nu\prime} \left( k_{||}, -i\frac{\partial}{\partial z} \right) + V(z)\delta_{\nu\nu\prime} + H^{\sigma}_{\nu\nu\prime} \right] g^{(\nu\prime)}_{m}(k_{||}, z)$$
$$= E^{\sigma}_{m}(k_{||})g^{(\nu)}_{m}(k_{||}, z)$$

We solve the above equation using the Finite Element Method (FEM) [12]. The advantage of FEM as a numerical technique over FDM is that it can utilize a nonuniform mesh, hence the energy eigenstates and wavefunctions of arbitrary shaped geometries with wide range of lateral dimensions can be analyzed accurately. The linear electronic suscenptibility  $\chi(\omega)$  of a strained quantum well laser can be derived using density matrix formalism [11].

$$\epsilon_{0}\chi(\omega) = \frac{1}{V} \sum_{\sigma,\eta} \sum_{l,m} \sum_{k_{||}} \left| \hat{\epsilon} \cdot \boldsymbol{M}_{lm}^{\sigma\eta}(\boldsymbol{k}_{||}) \right|^{2} \\ \sim \frac{f_{c}^{l} - f_{h,\sigma}^{m}}{E_{l}^{c}(\boldsymbol{k}_{||}) - E_{m}^{h,\sigma}(\boldsymbol{k}_{||} + E_{G} - \hbar\omega - i\eta/\tau_{in})}$$

 $\eta$  is the electron spin state, l and m are the subband indices for conduction and valence bands, respectively,  $\hat{\epsilon}$ is the polarization vector of the optical field,  $\epsilon_0$  is the permittivity of free space,  $M_{lm}^{\sigma\eta}$  is the optical dipole matrix element between the *l*th subband in the conduction band with a spin state  $\eta$  and the *m*th subband in the valence bands,  $f_c$  and  $f_v$  are the Fermi-Dirac occupation Yong Sang CHO et al.

$$g(\omega) = rac{\omega\mu c}{n_r} {
m Im} \epsilon_0 \chi(\omega)$$

where  $\mu$  is the permeability,  $n_r$  is the refractive index, and c is the speed of light in free space.

### III. RESULTS

We chose the structure shown in [14] as a reference with which in our results are compared. The structure is consisted of a 153 Å In<sub>0.53</sub>Ga<sub>0.47</sub>As well and In<sub>0.72</sub>Ga<sub>0.72</sub>As<sub>0.61</sub>P<sub>0.39</sub> barriers, which are lattice matched to InP. In the middle of the QW, a GaAs layer with 3-ML (9 Å) thickness is located. The valence band offsets due to the strain deformation potentials of the  $\delta$ -strained QW are shown in Fig. 1. The results were obtained from the model solid theory [10]. As in Fig. 1, the energy level of the light-hole band (dotted line) is much higher than that of the heavy-hole band (solid line) in the  $\delta$ -strained layer. This means that the  $\delta$ -strained layer confines heavy holes more strong than light holes. As a result, the quantization levels of the heavy hole states will be lowered whereas those of ligh hole sates will be rased. In Figure 3, the E-k dispersion relations of the QW with one GaAs  $\delta$ -strained layer are presented. The E-k dispersion relations of QW without the  $\delta$ -strained layer are also presented for comparison and such results are in elesewhere [11,14]. We labeled each subband as HH or LH according to their nature at  $k_{\parallel} = 0$  where the heavy hole and the light hole are decoupled. Compared to the QW without the  $\delta$ -strained layer, the first subband of the valence band is changed from heavy-hole band to light-hole



Fig. 1. The valence band offsets due to the strain deformation potentials in the  $\delta$ -strained layer. The solid line is for heavy hole band and the dashed line is for light hole band.



Fig. 2. The *E-k* dispersion relations of 153 Å thickness  $In_{0.53}Ga_{0.47}As/InGaAsP$  quantum well obtained by FEM. Each subband was labeled as HH or LH according to their nature at zone center.



Fig. 3. The *E-k* dispersion relations of 153 Å thickness QW with one GaAs  $\delta$ -strained layer obtained by FEM.

band as expected and presented in the reference [14]. On the other hand, the  $\delta$ -strained QW shows not only serious band mixing at  $k_{||} = 0$  but also the negative effective mass. Although the almost isotropic optical gain can be achieved in this condition, in the point of view of efficiency and reliability, the heavy band mixing effects are undesirable in most optical devices. These adverse aspects of the  $\delta$ -strained QW structure can be improved by introducing more  $\delta$ -strained layers to the proper posi-



Fig. 4. Optical gain characteristics of QW with one GaAs  $\delta$ -strained layer as a function of the locations of the  $\delta$ -strained layer. The solid lines are for TE mode and the dashed lines are for TM mode gains. The first simulation is shown as (1) and the third as (3) in the figure.

tions. To clarify the main contributions of the  $\delta$ -strained layer, we calculated with varying the positions of the  $\delta$ strained layer from the center to one of the side of the QW which is presented in Fig. 1. We presented in Fig. 4 the TE mode and the TM mode gain with solid lines and dashed lines, respectively. The first simulation is shown as (1) and the third as (3) in the figure. The step size to move the  $\delta$ -strained layer is 24 Å. As we move the  $\delta$ -strained layer with the step size, the TE and the TM mode gain are increased simultaneously. But, the ratio of TE and TM mode gain is reduced. Because the perturbation is reduced as the  $\delta$ -strained layer is moved, the resolved band mixing reduces the density of states in the valence bands. From the above results, we can infer that the main contribution of the  $\delta$ -strained layer is not merely introducing the strain effects to QW but also perturbing the wave functions of the valence bands.

We have also performed the comparison between our calculations and the results in the reference [14], which



Fig. 5. The *E-k* dispersion relations of QW with one GaAs  $\delta$ -strained layer obtained by FEM and by TB model. The solid lines are obtained by FEM and the dashed lines are obtained by TB model.

are obtained from the tight-binding method. As in Fig. 5, the results are qualitatively similar expect the band mixing at  $k_{\parallel} = 0$  where the heavy hole and light hole solutions are decoupled in the  $k \cdot p$  theory. Further deviations as  $k_{||}$  increases can be regarded as unimportant, which are believed due to different material parameters. Since heavy band mixing is not desirable in most cases, if we avoid the situations of heavy band mixing, the  $k \cdot p$ method based on the EFA is still applicable to complicated structures like  $\delta$ -strained QW. With the assumption that the  $k \cdot p$  method based on the EFA still applicable, we presented the results of QW with three  $\delta$ -strained layers (solid lines) and with one  $\delta$ -strained layer (dashed lines) in Fig. 6. We reduced the Ga composition to 0.8. The results show clearly that the heavy band mixing is much reduced compared to that of one  $\delta$ -strained layer. The optical gain characteristics of the two structures are presented in Fig. 7 for comparison. The TM mode gain is much enhanced than that of one  $\delta$ -strained case whereas the TE mode gain is slightly decreased because of the resolved band mixing and more perturbations due to  $\delta$ strain. This can improve the performance of the devices based on such QW. As in the case of QW with one  $\delta$ strained layers, the positions of the  $\delta$ -strained layers will change the band structures and the optical gain profiles. The most important merits of  $\delta$ -strained QW over conventional tensile strained QW is that one can achieve the TM mode dominant QW structure at 1.55  $\mu$ m with current growth thechnologies more easily. In our calculation, the well with of 1 % tensile strained QW must exceed 180 Å to obtain the peak wavelength of 1.55  $\mu$ m.



Fig. 6. The *E-k* dispersion relations of QW with one and three  $\delta$ -strained layers obtained by FEM. The solid lines are of QW with three  $\delta$ -strained layers and the dashed lines are of QW with one  $\delta$ -strained layer.





Fig. 7. Optical gain characteristics of QW with one and three  $\delta$ -strained layers. The solid lines are TE mode and the dashed lines are TM mode gains.

### **IV. SUMMARY**

In this paper, a inite element analysis on the QW with  $\delta$ -strained alyers has been performed. The main contribution of  $\delta$ -strained layers on a lattice matched QW is that they confine LH stronger than HH in the QW so as to change the first subband of the valance bands to LH band. For the widely known limitations of  $k \cdot p$  method, our results showed same trend with a former trial which were obtained with the TB model. We also showed that the heavy band mixing was vanished in the case of QW with three  $\delta$ -strained layers. It migth be though that the QW with  $\delta$ -strained layers can be applicable to optical devices of which polarization insensitivity is critical such as SOA and optical modulators.

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